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Potts Compound Markovian Texture Model

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Abstract

This paper describes a novel multispectral parametric compound Markov random field model for texture synthesis. The proposed compound Markov random field model connects a parametric control random field represented by a hierarchical Potts Markov random field model with analytically solvable wide-sense Markovian representation for single regions. The compound random field synthesis combines the modified fast Swendsen-Wang Markov Chain Monte Carlo sampling of the hierarchical Potts MRF part with the fast and analytical synthesis of single regional MRFs.

1. Introduction

Physically correct and convincing virtual models require not only detailed 3D shapes accorded with the captured scene, but also object surfaces covered with realistic nature-like surface material textures to enhance realism in virtual scenes. The primary purpose of any synthetic texture approach is to reproduce and enlarge a given measured texture image so that ideally both natural and synthetic texture will be visually indiscernible. BTF function is represented by thousands of measurements (images) per material sample, thus its modelling prerequisite is simultaneously also significant compression capability[4].

Compound random field models (CMRF) consist of several sub-models each having different characteristics along with an underlying structure model which controls transitions between these sub models [8]. CMRF models were successfully applied to image restoration [1, 8, 10], segmentation [15], or modeling [7], however these models always require demanding numerical solutions with all their well known drawbacks. The exceptional CMRF [7] model allows analytical synthesis at the cost of slightly compromised compression rate.

We propose a hierarchical Potts CMRF^{P3AR} model which combines two types of parametric Markov random field (MRF) models. One model can be analytically solved, while the other MRF can use exceptionally fast iterative Swendsen-Wang Markov Chain Monte Carlo (MCMC) sampling for its synthesis.

2. Compound Markov Model

Let us denote a multiindex $r = (r_1, r_2), r \in I$, where I is a discrete 2-dimensional rectangular lattice and r_1 is the row and r_2 the column index, respectively. $X_r \in \{1, 2, \ldots, K\}$ is a random variable with natural number value (a positive integer), Y_r is multispectral pixel at location r and $Y_{r,j} \in \mathcal{R}$ is its j-th spectral plane component. Both random fields (X, Y) are indexed on the same lattice I. Let us assume that each multispectral observed texture $\tilde{Y}$ (composed of d spectral planes e.g. $d = 3$ for colour textures) can be modelled by a compound Markov random field model, where the principal Markov random field (MRF) X controls switching to a regional local MRF model $Y = \bigcup_{i=1}^{K} {}^{i}Y$. Single K regional submodels ${}^{i}Y$ are defined on their corresponding lattice subsets ${}^{i}I$, ${}^{i}I \cap {}^{j}I = \emptyset \quad \forall i \neq j$ and they are of the same MRF type. They differ only in their contextual support sets ${}^{i}I_r$ and corresponding parameters sets ${}^{i}\theta$. The CMRF^{P3AR} model has posterior probability

$$P(X, Y \,|\, \tilde{Y}) = P(Y \,|\, X, \tilde{Y}) P(X \,|\, \tilde{Y}) \qquad (1)$$

and the corresponding optimal MAP solution is:

$$(\hat{X}, \hat{Y}) = \arg \max_{X \in \Omega_X, Y \in \Omega_Y} P(Y \,|\, X, \tilde{Y})\, P(X \,|\, \tilde{Y}) \ ,$$

where Ω_X, Ω_Y are corresponding configuration spaces for random fields (X, Y).

To avoid iterative MCMC MAP solution, we propose the following two step approximation:

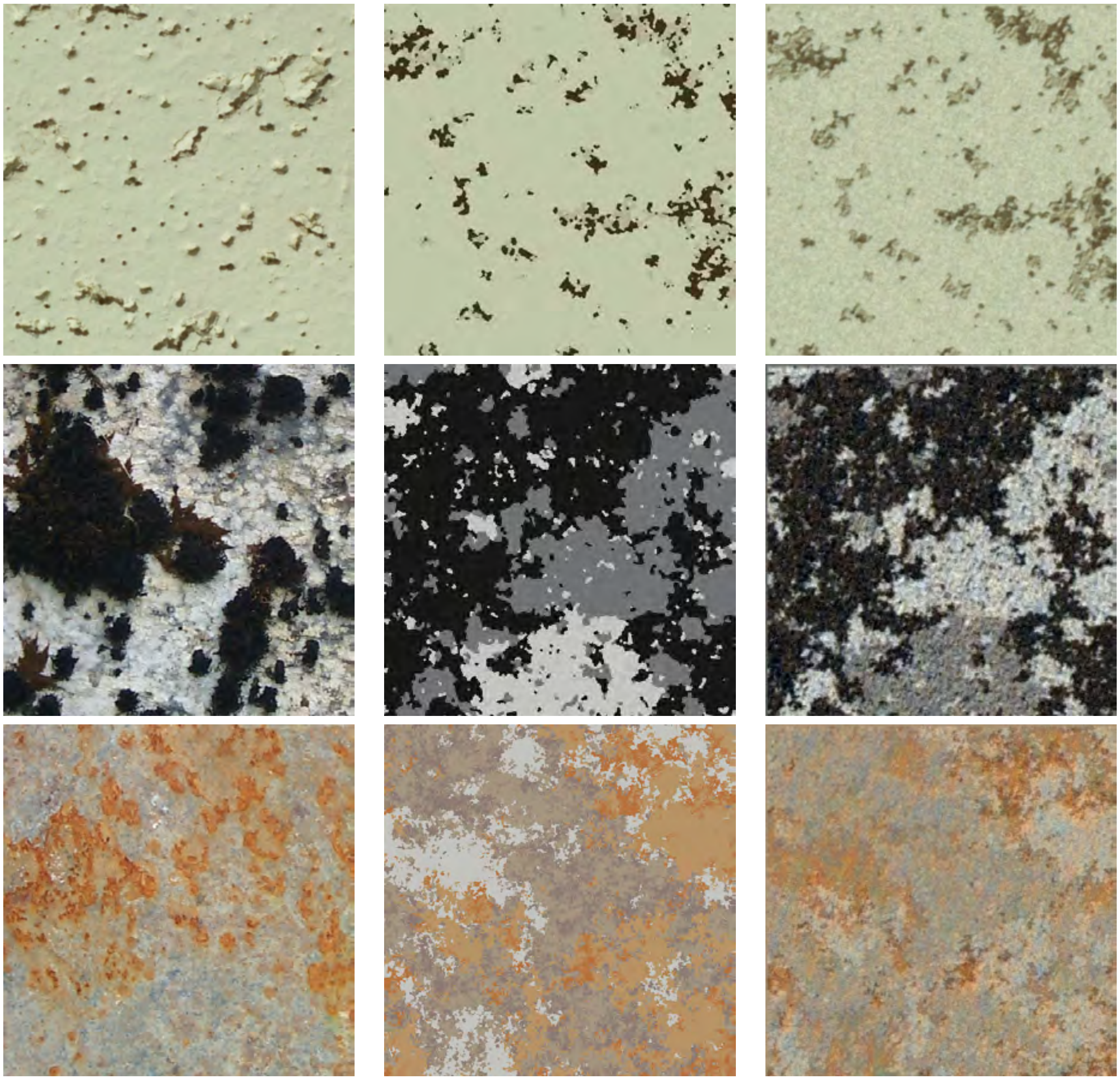

Figure 1. A ceiling panel texture measurement, its synthetic control field, and the final synthetic CMRF^{P3AR} model texture (upper row). The corresponding lichen and rusty plate synthetic texture results are in the subsequent rows.

$$(\breve{X}) = \arg\max_{X \in \Omega_X} P(X \mid \tilde{Y}) \ , \qquad (2)$$

$$(\breve{Y}) = \arg\max_{Y \in \Omega_Y} P(Y \mid \breve{X}, \tilde{Y}) \ . \qquad (3)$$

This approximation significantly simplifies CMRF^{P3AR} estimation because it allows to take advantage of simple analytical estimation of regional MRF models in (3).

2.1. Region Switching Markov Model

The principal MRF $(P(X \mid \tilde{Y}))$ is represented by a flexible $K-$state Potts random field [12, 14].

The learning control random field $\breve{X}$ is estimated from the target texture using simple K-means clustering of $\tilde{Y}$ in the RGB colour space into predefined number of K classes, where cluster indices are $\breve{X}_r \quad \forall r \in I$ estimates. The number of classes K can be estimated using the Kullback-Leibler divergence and considering sufficient amount of data necessary to reliably estimate all local Markovian models. The resulting thematic control map $\breve{X}$ is represented by the hierarchical two-scale Potts model

$$\breve{X}^{(a)} = \frac{1}{Z^{(a)}} \exp\left\{ -\beta^{(a)} \sum_{s \in I_r} \delta_{X_r^{(a)} X_s^{(a)}} \right\} \qquad (4)$$

where Z is the appropriate normalizing constant and $\delta()$ is the Kronecker delta function. The rough scale upper level Potts model ($a = 1$) regions are further elaborated with the detailed fine scale level ($a = 2$) Potts model which models the corresponding sub-regions in each upper level region. The parameter $\beta^{(a)}$ for both level models is estimated using an iterative estimator which starts from the upper β limit ($\beta_{\max}$) and adjusts (decreases or increases) its value until the Potts model regions have similar parameters (average inscribed squared region size and/or the region's perimeter) with the target texture switching field. This iterative estimator gives more resembling results with the target texture than the alternative maximum pseudo-likelihood method [9]. The corresponding Potts models are synthesized using the fast Swendsen-Wang sampling method [13].

2.2. Local Markov Models

Local i-th texture region (not necessarily continuous) is represented by the adaptive 3D causal autoregressive (3DCAR) random field model [3, 5] because this model can be analytically estimated as well as synthesised. Alternatively we could use spectrally decorrelated 2DCAR or 2D or 3D Gaussian Markov random field (GMRF) models [2, 6]. All these models allows analytical synthesis (see [2] for the corresponding conditions) and they can be unified in the following matrix equation form (i-th model index is further omitted to simplify notation):

$$Y_r = \gamma\, Z_r + \epsilon_r \ , \qquad (5)$$

where

$$Z_r = [Y_{r-s}^T : \forall s \in I_r]^T \qquad (6)$$

is the $\eta\, d \times 1$ data vector with multiindices r, s, t, $\gamma = [A_1, \ldots, A_\eta]$ is the $d \times d\, \eta$ unknown parameter matrix with sub-matrices A_s. In the case of d 2D CAR / GMRF models stacked into the model equation (5) the parameter matrices A_s are diagonal otherwise they are full matrices for general 3DCAR models [5]. The model functional contextual neighbour index shift set is denoted I_r and $\eta = cardinality(I_r)$. GMRF and CAR models mutually differ in the correlation structure of the driving noise ϵ_r (5) and in the topology of the contextual neighbourhood I_r (see [2] for details). As a consequence, all CAR model statistics can be efficiently estimated analytically [3] while the GMRF statistics estimates require either numerical evaluation or some approximation ([2]).

Given the known 3DCAR process history $Y^{(t-1)} = \{Y_{t-1}, Y_{t-2}, \ldots, Y_1, Z_t, Z_{t-1}, \ldots, Z_1\}$ the parameter estimation $\hat{\gamma}$ can be accomplished using fast, numerically robust and recursive statistics [3]:

$$\begin{aligned}
\hat{\gamma}_{t-1}^T &= V_{zz(t-1)}^{-1} V_{zy(t-1)} \ , \\
V_{t-1} &= \tilde{V}_{t-1} + V_0 \ , \\
\tilde{V}_{t-1} &= \begin{pmatrix} \sum_{u=1}^{t-1} Y_u Y_u^T & \sum_{u=1}^{t-1} Y_u Z_u^T \\ \sum_{u=1}^{t-1} Z_u Y_u^T & \sum_{u=1}^{t-1} Z_u Z_u^T \end{pmatrix} \\
&= \begin{pmatrix} \tilde{V}_{yy(t-1)} & \tilde{V}_{zy(t-1)}^T \\ \tilde{V}_{zy(t-1)} & \tilde{V}_{zz(t-1)} \end{pmatrix} \ , \\
\lambda_{t-1} &= V_{yy(t-1)} - V_{zy(t-1)}^T V_{zz(t-1)}^{-1} V_{zy(t-1)} \ ,
\end{aligned}$$

where V_0 is a positive definite matrix (see [3]). Although, an optimal causal (for (2D/3D)CAR models) functional contextual neighbourhood I_r can be solved analytically by a straightforward generalisation of the Bayesian estimate in [3], we use faster approximation which does not need to evaluate statistics for all possible I_r configurations. This approximation is based on large spatial correlations. We start from the causal part of a hierarchical non-causal neighbourhood and neighbours locations corresponding to spatial correlations larger than a specified threshold (> 0.6) are selected. The i-th model synthesis is simple direct application of (5) for both 2DCAR or 3DCAR models. GMRF models synthesis requires one FFT transformation at best [2]. 3D CAR / GMRF models provide better spectral modelling quality than the alternative spectrally decorrelated 2D models for motley textures at the cost of small increase of number of parameters to be stored.

3. Results

We have tested the presented novel CMRF^{P3AR} model on selected natural colour textures from our extensive texture database (http://mosaic.utia.cas.cz, Fig.1-lichen, rusty plate), which currently contains over 1500 colour textures, and on selected measurement from the University of Bonn [11] (Fig.1-ceiling). Examples on Figs.1 use six level control field ($K = 6$) and causal neighbourhood derived from the 4th order hierarchical contextual neighbourhood.

Other tested textures were either natural materials, such as lichen, clouds, bark, or stone , or selected manmade surfaces (ceiling panel, rusty plate). These resulting synthetic complex textures have generally better visual quality (there is no any reliable analytical quality measure) than textures synthesised using our previously published [2, 6, 5, 4] simpler MRF models. Synthetic multispectral textures are mostly surprisingly good for this fully automatic CMRF^{P3AR} algorithm. Obviously there is no universally optimal texture modelling algorithm and also the presented method will not produce good quality regular textures such as most textile or knitted wool textures.

The model can be easily generalized also for complex bidirectional texture function (BTF) models as it is demonstrated on one selected ceiling panel BTF measurement (zero elevation and azimuthal viewing angles). The full BTF-CMRF^{P3AR} variant of the presented model uses similar fundamental flowchart with our Markovian BTF model [4] (i.e. BTF space intrinsic dimensionality estimation, BTF space segmentation, BTF subspace MRF model estimation, subspace MRF model synthesis and interpolation of unmeasured BTF space parts) but allows to avoid its range map estimation, range map modelling and displacement filter steps, respectively. The presented CMRF^{P3AR} model needs to store only tens of parameters and thus it is capable to reach huge compression rate relative to the alternative sampling or hybrid based texture synthesis approaches.

4. Conclusions

The presented CMRF^{P3AR} method shows good performance on the selected class of tested real-world materials. It offers large data compression ratio (only tens of parameters per single multispectral texture) easy simulation and exceptionally fast seamless synthesis of any required texture size. The method can be easily generalised for BTF texture modelling or for colour or BTF texture editing by combining estimated local models from several target textures. The method even allows to synthesize unseen (unmeasured) textural data by changing several selected parameters.

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